Platform Competition with Multihoming on Both Sides: Subsidize or Not?

— extended abstract —

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1 Introduction

The literature on competition between two-sided platforms, going back to Armstrong (2006), mostly ignores the case of multihoming agents on both sides. Most analyses either consider single homing by participants on both sides of a platform, or allow multihoming by participants on one side of the platform while they impose single homing on the other side. These models also typically assume full coverage on both sides, i.e. that all agents participate on at least one platform. The few analyses allowing for multihoming on both sides tend to apply in specialized settings, like Caillaud and Jullien (2003) who consider multihoming in a matching setting, where multihoming agents get additional chances at being matched.

The argument in favor of not considering multihoming on both sides of the market is that if one side of the market fully multihomes, there is no benefit to allowing the other side of the market to also multihome; as all possible pairs of agents could already connect with each other. This assumption is limiting, however, if each side of the market only partially multihomes, in which case multihoming on the other side does generate new potential interactions between agents on the two sides of the market. It is also limiting if the platforms are differentiated, in which case two agents meeting on one platform would create different surplus than the same agents meeting on a different platform.

A major result in the two-sided platform literature is that there is interdependence between the two sides served by the same platform; meaning that lowering the price on one side can make the platform more competitive on the other side (without lowering its price there). The policy implication is that a platform may maximize its total profits by subsidizing one side, a typical example being that it may be optimal for Adobe to offer Acrobat Reader to consumers at a zero, or even a negative, price, in order to maximize its profits from the sale of the corresponding authoring tools.

In this project we develop a model for platform competition in a differentiated setting (a Hoteling line), which is similar to other models in the literature. However we focus on the case where at least some agents on each side multihome, and we show that in that case the strategic interdependence between the two sides of the same platform may be of lesser importance, or even not be present at all, compared to models that assume single-homing on at least one side of the market. The implication is that when multihoming may be present on both sides of the market, the benefit of subsidizing one side (typically the one with higher price elasticity) is diminished or may not be present at all.

Our results suggest that we need to be wary of overstating the importance of the interdependence even when it does exist, or we risk missing the important drivers of the market equilibrium, and give wrong strategic advice. This is a significant departure from the two-sided platform literature, which identifies interdependence between the two sides served by the same platform, and has policy implications about a platform's ability to maximize total profits by subsidizing one side.

2 Model

We are analyzing following model:

- two-sided Hoteling competition

- side X, $x \sim U[0,1]$
- side Y, $y \sim U[0,Y]$, $Y \gtreqless 1$
- two platforms, A and B, located at 0 and 1 (Y)
- platforms charge p_i, $i = A, B$ on side X and r_i on side Y
- user located at x on side X (resp. y on Y) receives following utility from joining platform $i = A, B$

$$\begin{aligned} u(x;A) =& A_x + \alpha y_A - p_A - zx \\ u(x;B) =& B_x + \alpha(Y - y_B) - p_B - z(1-x) \\ u(y;A) =& A_y + \beta x_A - r_A - qy \\ u(y;B) =& B_y + \beta(1 - x_B) - r_B q(Y - y) \end{aligned} \tag{1}$$

  where y_A etc is the number of agents on side Y that participate in platform A.

- A_x, A_y — stand-alone values of platform A for X and Y users
- α, β — "network effect" of the other side on side X and Y
- z, q — "transportation cost": loss of utility due to preference mis-match, or set-up costs

The setup of this model is shown in Figure 1.

3 Single-homing benchmark

We begin by analyzing as a benchmark the case with full coverage and single-homing, as is typical in most of the literature on platform competition.

- imposing single-homing and full coverage: $x_A = x_B = \tilde{x}$ s.t. $u(\tilde{x};A) = u(\tilde{x};B)$ and $y_A = y_B = \tilde{y}$

$$\tilde{x} = \frac{z + A_x - B_x + 2\alpha\tilde{y} - \alpha Y - p_A + p_B}{2z}$$

$$\tilde{y} = \frac{qY + A_y - B_y + 2\beta\tilde{x} - \beta - r_A + r_B}{2q}$$

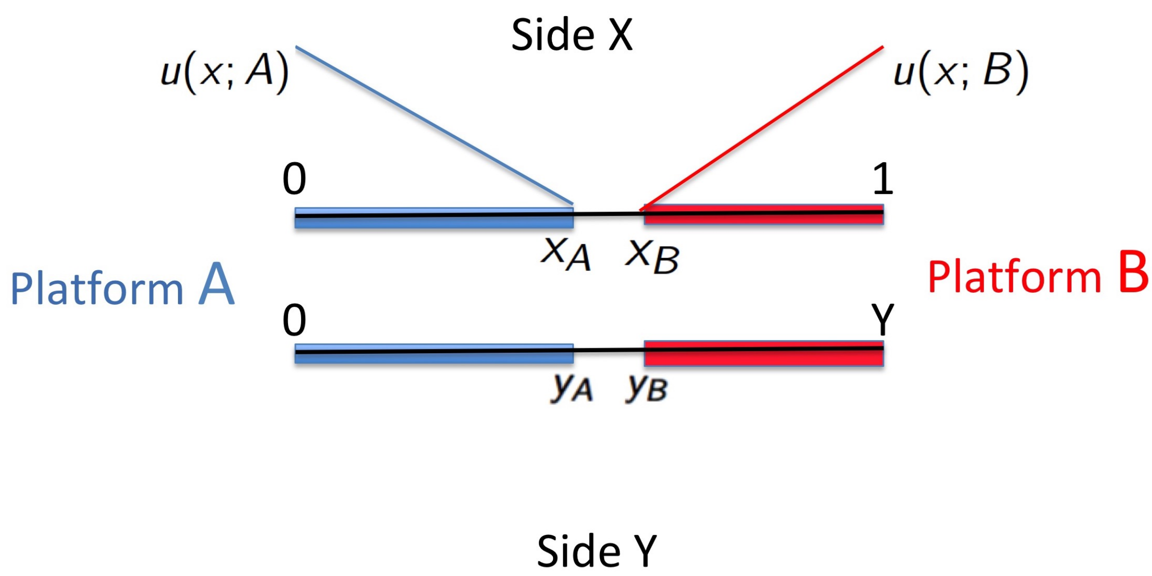


Figure 1: Setup

- $\tilde{x}$ depends on p_A, p_B and r_A, r_B
- interdependence for each platform between its prices on the two sides

4 Allowing for multihoming on both sides

We now allow for multihoming on both sides of a platform.

Utility when multihoming. We first characterize the utilities agents get when multihoming, $u(x; A\&B)$ and $u(y; A\&B)$.

The market coverage of platform A is given by x_A, and $1 - x_B$ is the market coverage of platform B. Multihoming on side X occurs when $x_A > x_B$. Multihoming on both sides occurs when $x_A > x_B$ and $y_A > y_B$. In such a case, an x agent who is multihoming may meet the same y agents on both platforms, as y agents are multihoming as well; and vice versa.

There are many possibilities for the functional form of $u(x; A\&B)$ and $u(y; A\&B)$. When two agents meet on two different platforms, they could receive the network benefit twice (once on each platform). We call that case "double-counting" of the network effects from the overlapping agents. At the other extreme, the agents may get the benefit only from meeting once, and no additional benefit from meeting again — "no double counting." In the intermediate case, meeting for the second time may yield partial additional network advantage — "partial double counting." Similarly we need to consider the benefit from intrinsic values of the two platforms when an agent is multihoming.

For the base case of our analysis, we assume double counting of intrinsic values, but no double counting of the network effect from overlapping agents. Even though X side users may meet some Y side users on both platforms, they only get the network benefit once. Therefore, joining both platforms when $y_A \geq y_B$ yields

$$u(x; A\&B) = A_x + B_x + \alpha Y - p_A - p_B - z$$

Utility of an agent joining A only, without having joined the other platform, is given by $u(x; A)$. Note that

$$u(x; A\&B) < u(x; A) + u(x; B) \text{ if } y_A > y_B$$

Similarly for $u(y; A\&B)$.

Decision to participate in both platforms. An agent will multihome when multihoming yields higher utility than joining only platform A, only platform B, or not joining either of the platforms.

Utility of an agent joining A without having joined the other platform is given by $u(x; A)$ as in (1). Therefore, the agent indifferent between joining platform A only and not joining any platform at all, $\bar{x}_A$, is characterized by $u(\bar{x}_A; A) = 0$, i.e.,

$$\bar{x}_A = \frac{A_x + \alpha\, y_A - p_A}{z} \tag{2}$$

Users $x < \bar{x}_A$ would prefer to join A while users $x > \bar{x}_A$ would prefer not to join at all (see Figure 2a). I.e., $\bar{x}_A$ would be the market captured by platform A if it was the only platform.

Similarly, given only the choice of platform B or no platform, all users $x > \bar{x}_B$ would prefer to join B, while $x < \bar{x}_B$ would not join (see Figure 2b), where

$$\bar{x}_B = 1 - \frac{B_x + \alpha\,(Y - y_B) - p_B}{z}$$

With multihoming, agents will consider joining A while they already participate in B. In such a case, user x's utility from joining A in addition to B is given by

$$u(x; A|B) = u(x; A\&B) - u(x; B)\,. \tag{3}$$

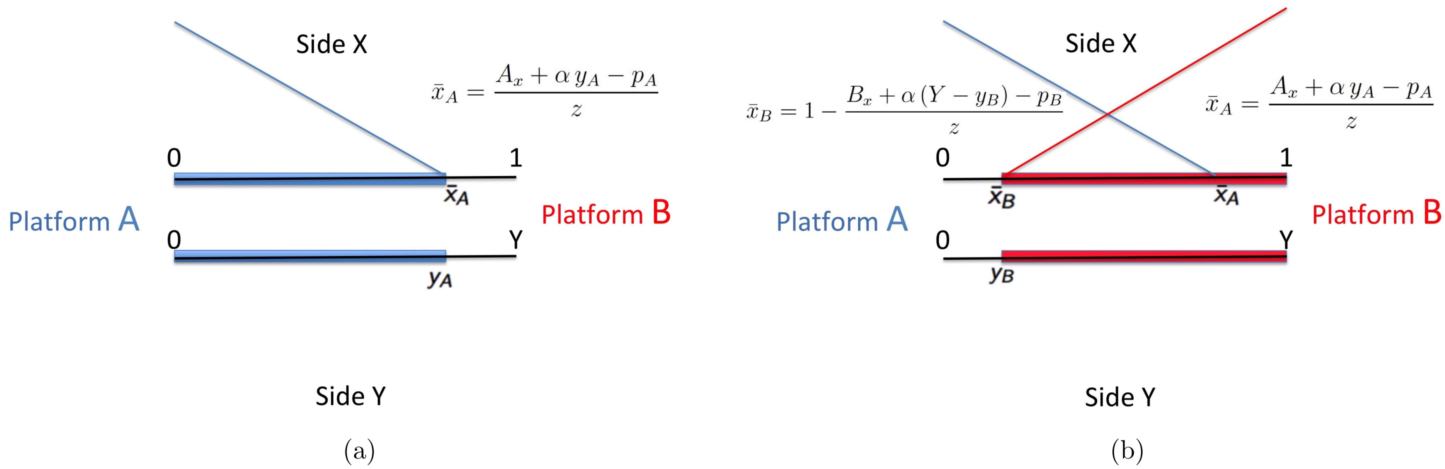


Figure 2

Note that if there is multihoming on side Y, this incremental utility $u(x; A|B)$ must be less than $u(x; A)$.[1] That means that some agents who might have joined A if no other platform was available, would not have joined A as the second platform. I.e., for some x, $u(x; A|B) < 0 < u(x; A)$. Thus, the actual market captured by platform A in the case of multihoming on both sides is smaller than $\bar{x}_A$.

To characterize the size of the market captured by platform A in the case of multihoming on both sides, we need to identify the agent who is indifferent between joining A in addition to B, and just staying with B only. This agent, denoted by $\hat{x}_A$, is characterized by $u(\hat{x}_A; A|B) = 0 \iff u(\hat{x}_A; A\&B) = u(\hat{x}_A; B)$.

$$\hat{x}_A = \frac{A_x + \alpha\, y_B - p_A}{z} \tag{4}$$

In the case of multihoming on both sides, platform A captures market of size $\hat{x}_A$ on side X. It is straightforward to note that since $y_A > y_B$, then $\hat{x}_A < \bar{x}_A$.

Notice the difference between formulas (2) and (4). The threshold $\bar{x}_A$ depends on the number of opposite-side agents available on the same platform, y_A. That is, it depends on pricing decision of the same platform A. But $\hat{x}_A$ depends on y_B, which depends on the pricing decision of *the other* platform.

Interestingly, platform A cannot make itself more appealing to agents on side X by increasing the number of Y agents it attracts. In fact, the attractiveness of platform A to

[1] Another thing to note is that (3) is valid no matter what we take for $u(x; A\&B)$, i.e., whether we double count intrinsic values and the network effect from overlapping platform members.

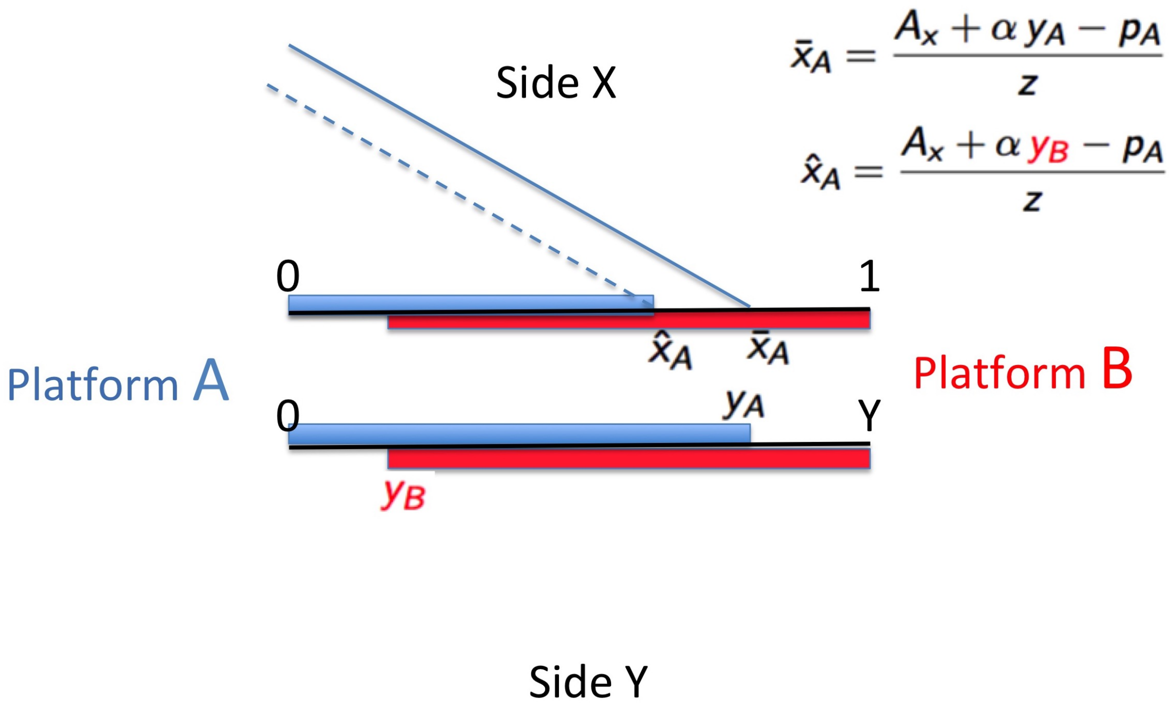


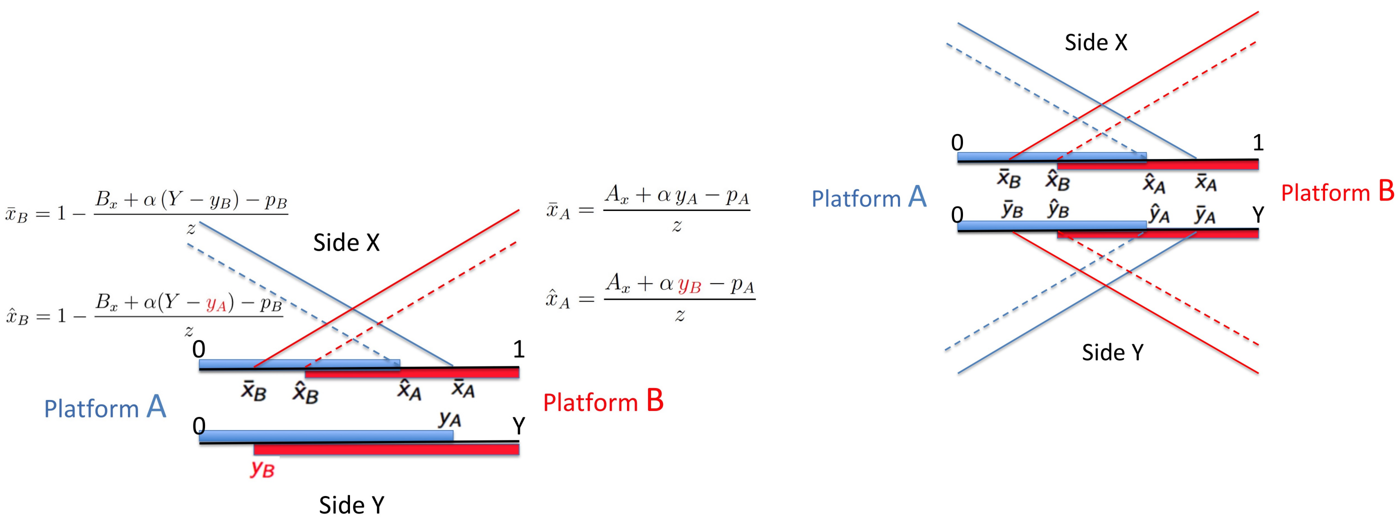
Figure 3: Market coverage of platform A when multihoming on both sides occurs, $\hat{x}_A$

agents on side X depends on side Y agents attracted by the other platform; this is because y_B represents the number of side Y agents that are *exclusive* to platform A. As platform B becomes more attractive to side Y agents, the number of such agents joining A exclusively decreases—even if A can increase its overall coverage of side Y. This lowers the attractiveness of joining A for the marginal side X agent, because the marginal side X agent is deciding whether to join A in addition to B, not whether to join either A or no platform at all; and the marginal side X agent already has access to these side Y agents on platform B.

More formally, the profit maximizing $\bar{x}^*_A$ depends on p_A and r_A, the two prices set by platform A. The profit maximizing $\hat{x}^*_A$, however, depends on p_A and r_B, but not on r_A. Thus $\hat{x}^*_A$, which is the relevant profit maximizing price on side X, does not depend on the price set by platform A on the Y side.

5 Equilibria with multihoming on both sides

The focus of our analysis in this section is to characterize equilibria where agents on both sides multihome, and especially equilibria where only some agents on both sides multihome, while others singlehome. We call it *partial multi-homing*.



(a) (b)

Figure 4: Participation decision with multihoming on both sides

Partial multihoming on both sides occurs in equilibrium when $\hat{x}_A > \hat{x}_B$ and $\hat{y}_A > \hat{y}_B$. Note also that in such a case: $x_A = \hat{x}_A$, $x_B = \hat{x}_B$, $y_A = \hat{y}_A$ and $y_B = \hat{y}_B$. Using the derivation of the previous section, we obtain

$$
\begin{aligned}
\hat{x}_A &= \frac{A_x + \alpha \hat{y}_B - p_A}{z} \\
\hat{x}_B &= 1 - \frac{B_x + \alpha(Y - \hat{y}_A) - p_B}{z} \\
\hat{y}_A &= \frac{A_y + \beta \hat{x}_B - r_A}{q} \\
\hat{y}_B &= Y - \frac{B_y + \beta(1 - \hat{x}_A) - r_B}{q}
\end{aligned}
\tag{5}
$$

Note the interaction between $\hat{x}_A$ and $\hat{y}_B$ (and therefore between p_A and r_B), but NOT between $\hat{x}_A$ and $\hat{y}_A$. That is, there is no strategic interaction between pricing on the two sides of the same platform.

Proposition 1 *If both sides multihome and there is no double counting of the network benefits from meeting the same other side agent on both platforms, there is no interdependence of prices on the two sides of the sane platform when maximizing profit. I.e., the profit maximizing p_i^* does not depend on r_i^*.*

Proof. This follows directly from FOC for profit maximization. Technically, r_i does not enter the FOC for maximizing profit with respect to p_i.

Corollary 1 *In the environment described in Proposition 1, it is never profitable for a platform to subsidize one side of the market, i.e., charge a negative price.*

The strategic interaction between $\hat{x}_A$ and $\hat{y}_B$ is fueled by the strength of the network effects). When network effects are strong, i.e., $\alpha\beta > q\,z$, then no pure strategy equilibrium with partial multihoming exists. However, there may exist equilibria with multihoming on both sides where thresholds are outside of the interval, even for strong network effects.

In the rest of the proposal we focus on the case where network effects are weaker than transportation costs, i.e., $\alpha\beta < q\,z$.

For $\alpha\beta < q\,z$, equilibrium with partial multihoming on both sides is characterized by

$$\hat{x}_A^* = \frac{q(A_x(2qz-\alpha\beta)-\alpha(B_yz-2qYz+z\beta+Y\alpha\beta))}{(qz-\alpha\beta)(4qz-\alpha\beta)}$$

$$\hat{y}_A^* = \frac{z(A_y(2qz-\alpha\beta)-\beta(B_xq-2qz+qY\alpha+\alpha\beta))}{(qz-\alpha\beta)(4qz-\alpha\beta)}$$

$$\hat{x}_B^* = \frac{2q^2z(2z-Y\alpha)+\alpha^2\beta^2+B_xq(-2qz+\alpha\beta)+q\alpha(A_yz-4z\beta+Y\alpha\beta)}{(qz-\alpha\beta)(4qz-\alpha\beta)}$$

$$\hat{y}_B^* = \frac{4q^2Yz^2+qz(A_x-2z-4Y\alpha)\beta+\alpha(z+Y\alpha)\beta^2+B_yz(-2qz+\alpha\beta)}{(qz-\alpha\beta)(4qz-\alpha\beta)}$$

And

$$p_A^* = -\frac{A_x(-2qz+\alpha\beta)+\alpha(B_y z - 2qYz + z\beta + Y\alpha\beta}{4qz-\alpha\beta}$$
$$r_A^* = -\frac{A_y(-2qz+\alpha\beta)+\beta(B_x q - 2qz + qY\alpha + \alpha\beta)}{4qz-\alpha\beta}$$
$$p_B^* = \frac{(B_x(2qz-\alpha\beta)-\alpha(A_y z - 2qYz + z\beta + Y\alpha\beta)}{4qz-\alpha\beta}$$
$$r_B^* = \frac{B_y(2qz-\alpha\beta)-\beta(A_x q - 2qz + qY\alpha + \alpha\beta)}{4qz-\alpha\beta}$$

Proposition 2 *For an appropriate range of parameters, there exists a pure strategy equilibrium with multihoming on both sides .*

Proof. For now, we prove the existence by providing a set of parameters for which such an equilibrium exists. We plan to derive soon an analytical formulation for the applicable range of parameters.

Consider the following parameter values: $\alpha = \beta = 0.1$; $q = z = 0.2$; $A_x = A_y = B_x = B_y = 0.26$; $Y = 1$. These values result in the following pure strategy equilibrium: $p_A = r_A = p_B = r_B = 0.12; x_A = y_A = 0.8; x_B = y_B = 0.2$ and profits $\pi_A = \pi_B = 0.192$. ■

Note that for other parameter values, different pure strategy equilibria are possible. For instance, equilibria may arise where the market on one or both sides is not fully covered, or where singlehoming arises endogenously.

6 Meeting the same agent on additional platforms yields incremental network benefit

So far, we have assumed that participants that overlap on multiple networks do not derive additional benefit from being able to meet on more than one platform; in other words there is no double counting of overlapping network members and no double counting of the corresponding network effects. Under this assumption, we have arrived at equilibrium conditions (5), which indicate that there is no interdependence between pricing decisions on the two sides by the same platform. It is a striking result, especially when compared with the single homing benchmark.

As we have noted in Section 4, there are many possibilities for how utility of multihomers is specified. Different specifications may be more suitable for different markets.

In this section, we generalize the analysis, by allowing partial double counting of the network effects in the utility of multihomers. That is, meeting the same agent on the second platform yields some incremental network benefit above the network benefit from meeting him on the first platform. As we will see, when the partial double counting is allowed, the interdependence in pricing of two sides by the same platform is present again. However, the strength of this interaction directly depends on the size of the double-counting. With very little double-counting, the interdependence is negligibly small.

Our aim in this analysis is to point out that while the interdependence plays primary role in environments with assumed single-homing, the interdependence is of lesser importance, and may disappear completely, when the platforms are competing in an environment where multihoming on both sides is possible.

Set up of the generalized problem. The measure of the multihoming agents on side Y is $y_A - y_B$. The utility of a multihoming agent x is

$$u(x; A\&B, \omega) = A_x + B_x + \alpha[Y + \omega(y_A - y_B)] - p_A - p_B - z\,,$$

where $\omega \in [0, 1]$ is the size of the double-counting. For $\omega = 0$, there is no double counting. In the special case of $\omega = 1$, $u(x; A\&B, \omega = 1) = u(x, A) + u(x, B)$. That is, the agent gets full double benefit from meeting a Y agent twice on the two platforms. In a way, it means that the two platforms provide different benefits, and there isn't much competition between them.

Now, even with ω, formula (3) is still valid

$$u(x; A|B, \omega) = u(x; A\&B, \omega) - u(x; B) = A_x + \alpha[\omega y_A + (1 - \omega)y_B] - p_A - z\,x\,.$$

And thus

$$\hat{x}_A(\omega) = \frac{A_x + \alpha[\omega y_A + (1 - \omega)y_B] - p_A}{z}$$

Based on the discussion at the end of Section 4, ω measures the interdependence between the price platform A sets on side Y, and how attractive it is to agents on side X. This interdependence ranges with the value of ω, and for small ω is close to negligible. With small ω, the price set by the other platform on side Y is much more important to determining $\hat{x}_A$.

Note also that for $\omega = 1$, $\hat{x}_A(\omega = 1) = \bar{x}_A$, while for $\omega < 1$, $\hat{x}_A(\omega) < \bar{x}_A$.

Proposition 3 *If there is some incremental network benefit of meeting the same other-side agent on both networks (”double counting” of common agents), the strength of the interdependence between the prices charged on the two sides by the same platform is determined by the strength of the above double counting.*

7 Empirical testing with data from credit card use by Canadian consumers and merchants

- Credit cards offer a well known example of two sided platforms, the two sides being the merchants and the consumers.

- Merchants typically accept several credit cards, thus multihoming is common in the merchant side of the platform.

- Most consumers also use several credit cards, and thus also multihome (in different degrees). While there is a clear benefit to a consumer of having a credit card accepted by a merchant they desire to do business with, there is typically little incremental benefit in possessing more than one credit card accepted by the merchant; thus the benefit of a consumer and a merchant “meeting” on multiple platforms is limited after the ability to transact is established (in our terminology, there is no “double counting” of the interaction benefit).

  To empirically test the predictions of our analysis, we draw on a unique data set from a surveys of Canadian consumers and merchants conducted by Bank of Canada in 2014 and 2017. Participants to the consumer survey were asked to report all credit cards they hold, and to identify all purchases they made over the course of three days, including which credit card they used for the credit card purchases. This data will allow us to estimate the extend of multihoming on the users side. The merchant survey asked participants whether they accept different credit cards and what are the fees they pay for each credit card they accept. We are in the process of analyzing the data to determine whether credit cards that face higher multihoming on both sides are less likely to offer subsidies, as predicted by our theoretical results.

8 Conclusion

We analyze platform competition when agents multihome on both sides.

- Literature mostly ignores this case – and derives results where the pricing interdependence between the sides is central

- Once we allow for multihoming on both sides, it is important to specify what is the utility of the multihoming users, who can also meet multihoming users on the other side — that is, they meet each other twice on the two platforms. Do they obtain the benefit of interaction twice, or only once?

- In the base model, we decide we look at the case where they obtain the benefit only once (no-double counting). For this specification, we show:

  - under certain conditions, equilibria exist with multihoming on both sides
  - when we have multihoming on both sides, the interdependence between the two sides plays out differently than under single-homing
  - specifically, there is no interdependence between the two sides of the same platform
  - optimal pricing for a platform on one side depends on the prices of the other platform only
  - thus, it is never optimal to subsidize the other side (no divide-and-conquer strategy)
  - that is very different from analyses imposing single-homing on one or both sides

While we start with the extreme specification where meeting the same agent for the second time brings no benefit, we also extend our analysis to more general formulation, where the agents can gain partial benefit from meeting each other the second time.

- In this case, the interdependence is present again. However, it strongly depends on the size of the benefit from the second meeting. If the partial benefit is very small, the interdependence is also negligible. At the same time, pricing of the other platform is a much more important factor.

In conclusion, we need to be wary of overstating the importance of the interdependence even when it does exist, or we risk missing the important drivers of the market equilibrium, and give wrong strategic advice.